Using Progressions and Learning Trajectories to Guide Intervention in Addition and Subtraction

Shannon Olson
Educational Consultant, President & Founder
Olson Educational Services, LLC

Utah Early Childhood Conference
March 17, 2023

To access the Nearpod link and other session materials, please go to:

shannonolson.com/uaeyc

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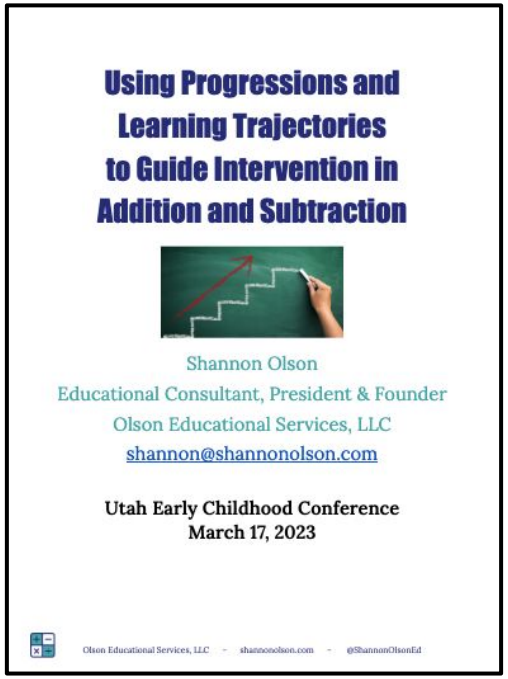
Utah Council of Teachers of Mathematics: March 17, 2023

Welcome

Setting Up for the Session



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Let's Get to Know Each Other: Who's in the room?

- Teachers
  - Preschool
  - Kindergarten
  - First grade
  - Second grade
  - Third grade
  - Upper grades
- Coaches
- Specialists
- Administrators
- Others

Let's Get to Know Each Other

Meet your neighbors

- Name
- Position
- District/School
- What you are excited to learn today

Learning Intentions:

– – –

- **anticipate** student thinking for **addition and subtraction**
- observe and describe **student thinking**
- understand **learning progressions** and **learning trajectories**
- **make connections** for **planning intervention**

Learning Intentions:

- **anticipate** student thinking for **addition and sub ion**
- observe and describe **student thinki**
- understand **learning progr ories**
- **make connectio**

What is your personal learning intention?

Agenda

- Welcome
- Mathematics Task
- Evidence of Student Thinking
- Learning Progressions
- Learning Trajectories
- Intervention Strategies
- Intervention in Action
- Closing and Reflection

Mathematics Task

What do you notice?

What do you wonder?

Adapted from Mathematics Georgia Standards of Excellence (GSE) K-5, 2022.

Video

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Solve the Problem

_ _ _

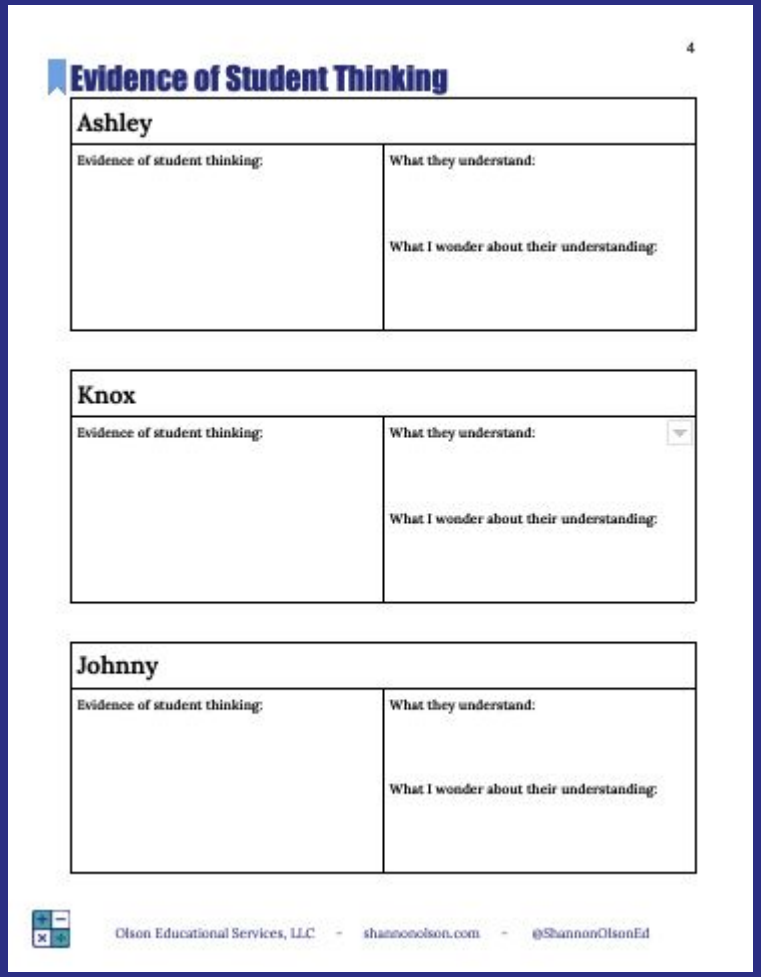
$$16 + 26$$

Anticipate Student Thinking

16 + 26

- How might students solve the problem?
- What strategies and representations would they use?
- What errors or misconceptions might they have?

Evidence of Student Thinking

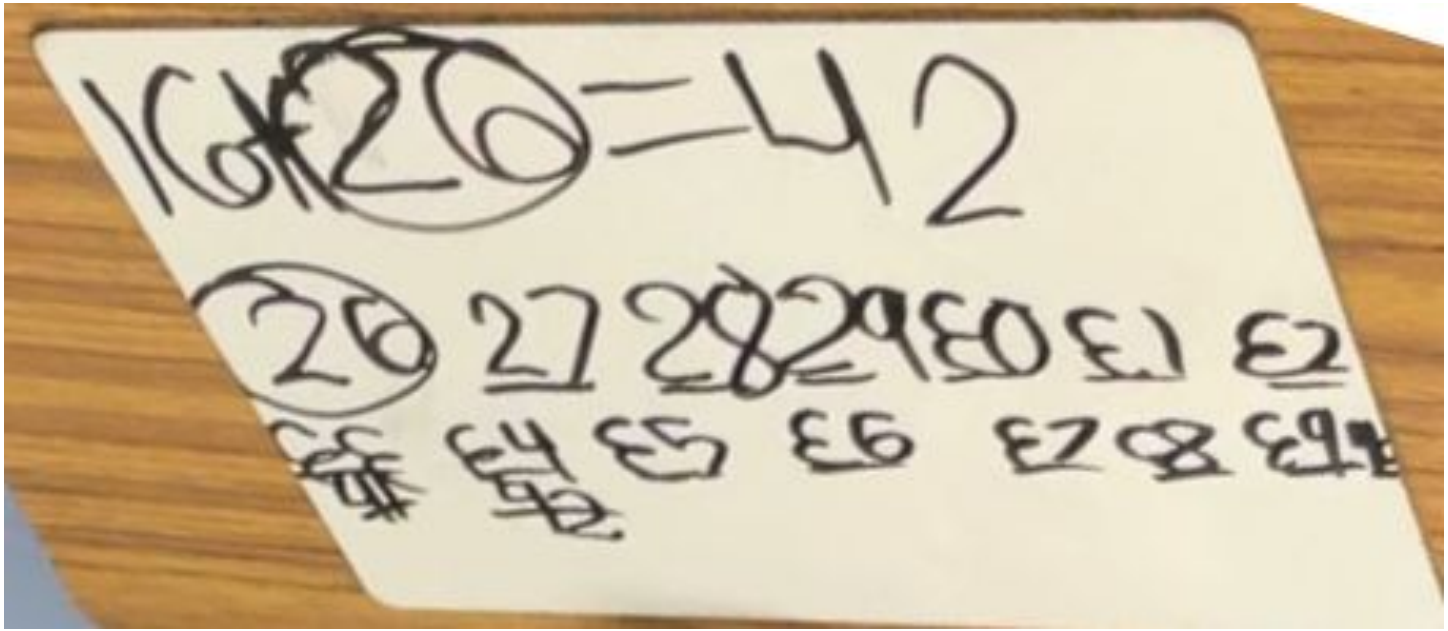


Evidence of student thinking:

What they understand:

What I wonder about their understanding:

Ashley



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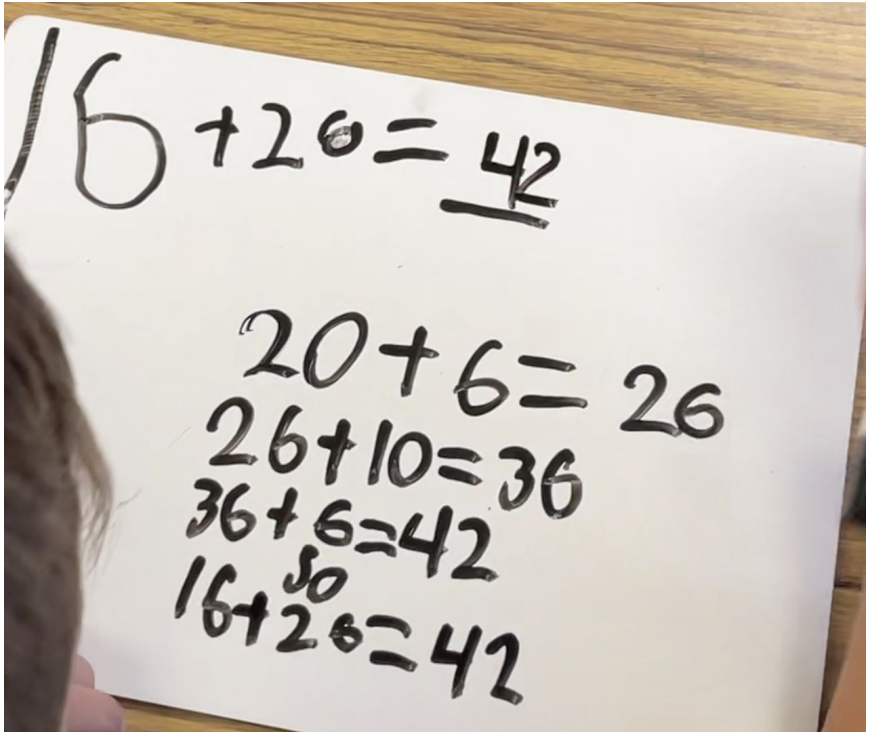
Knox



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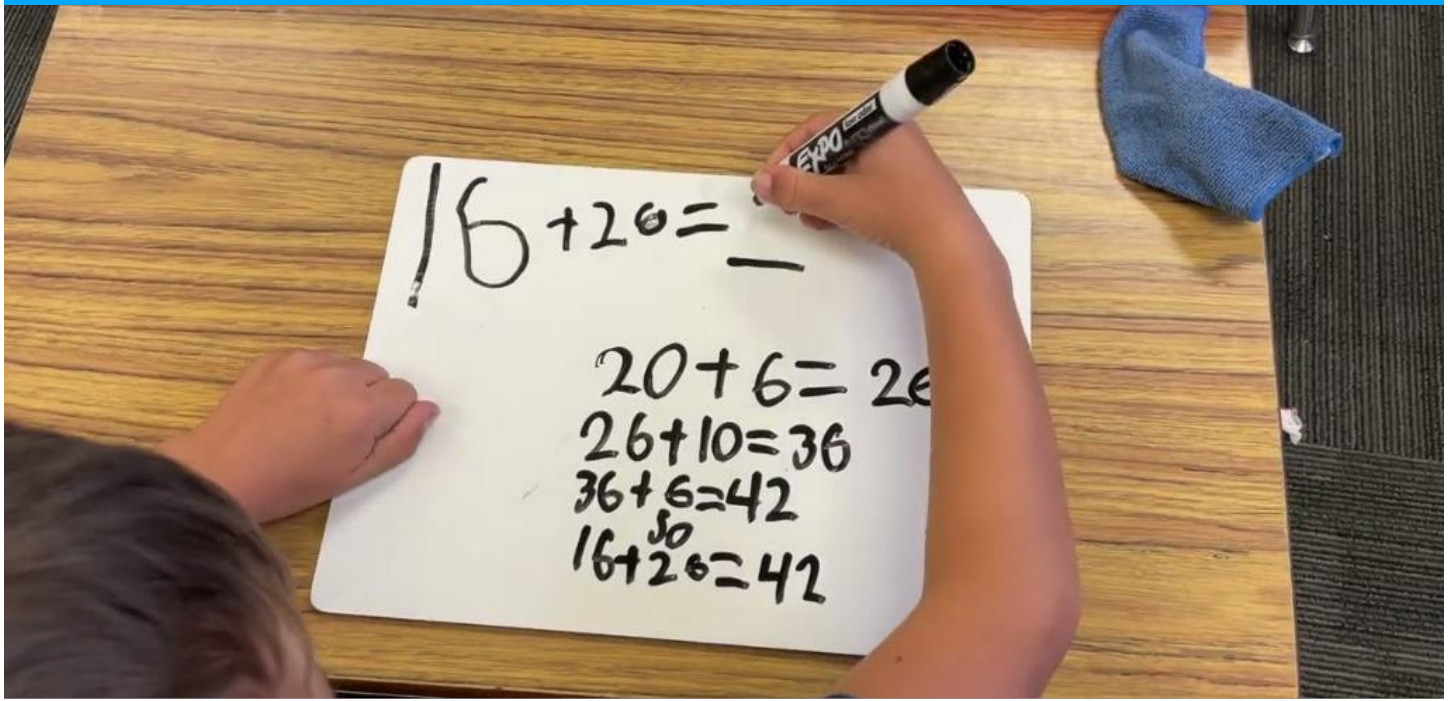

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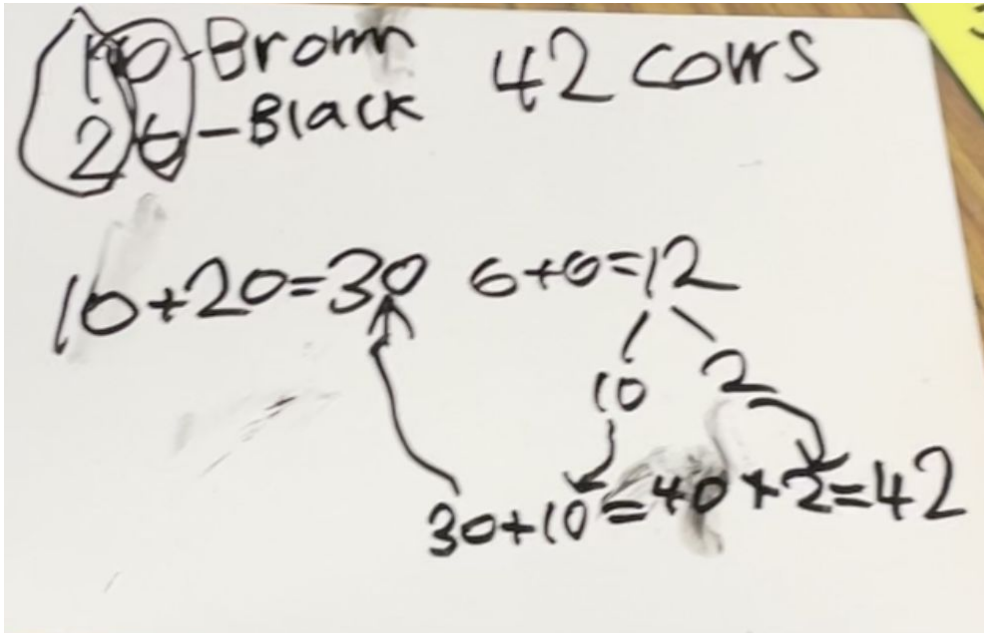
Johnny



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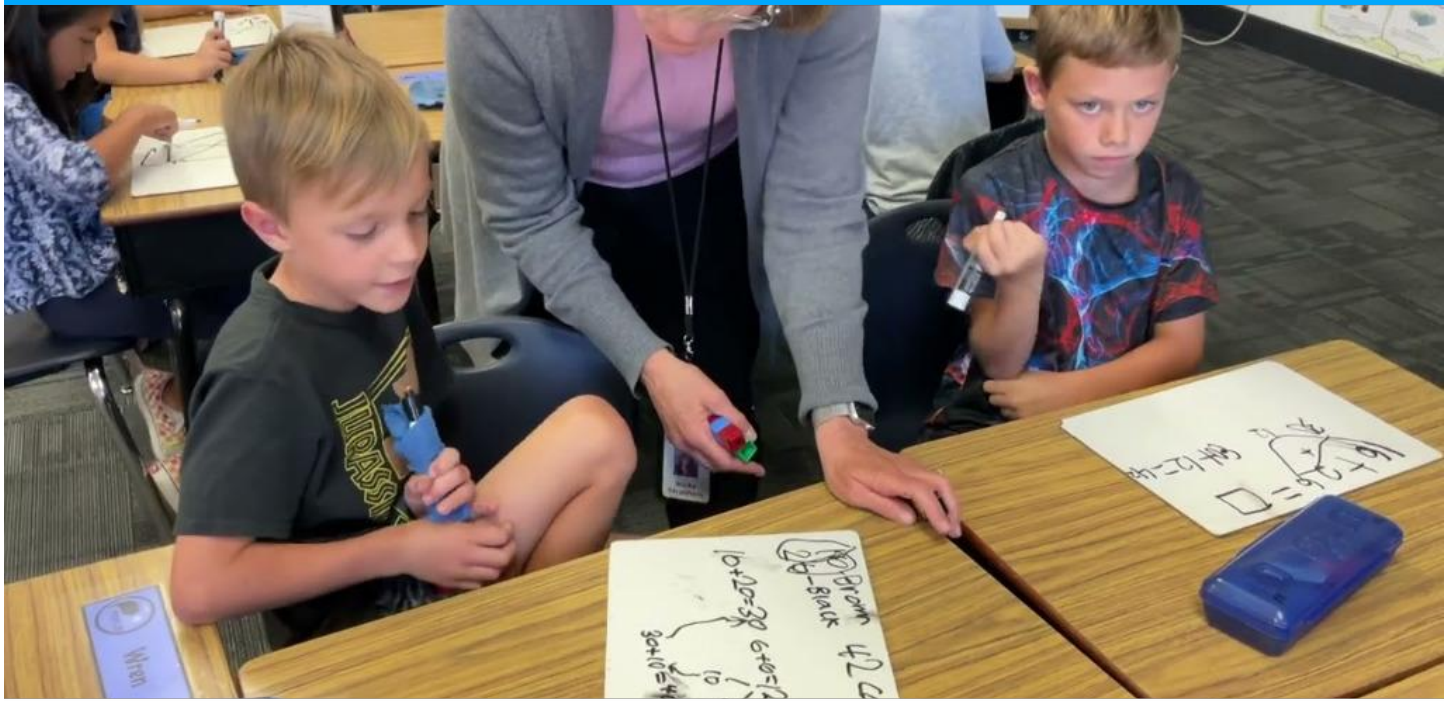

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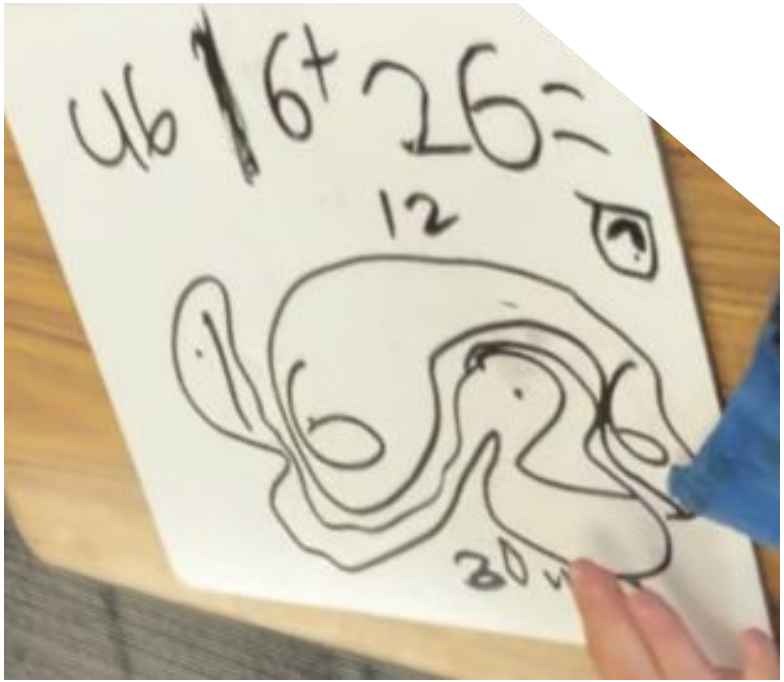
Wren



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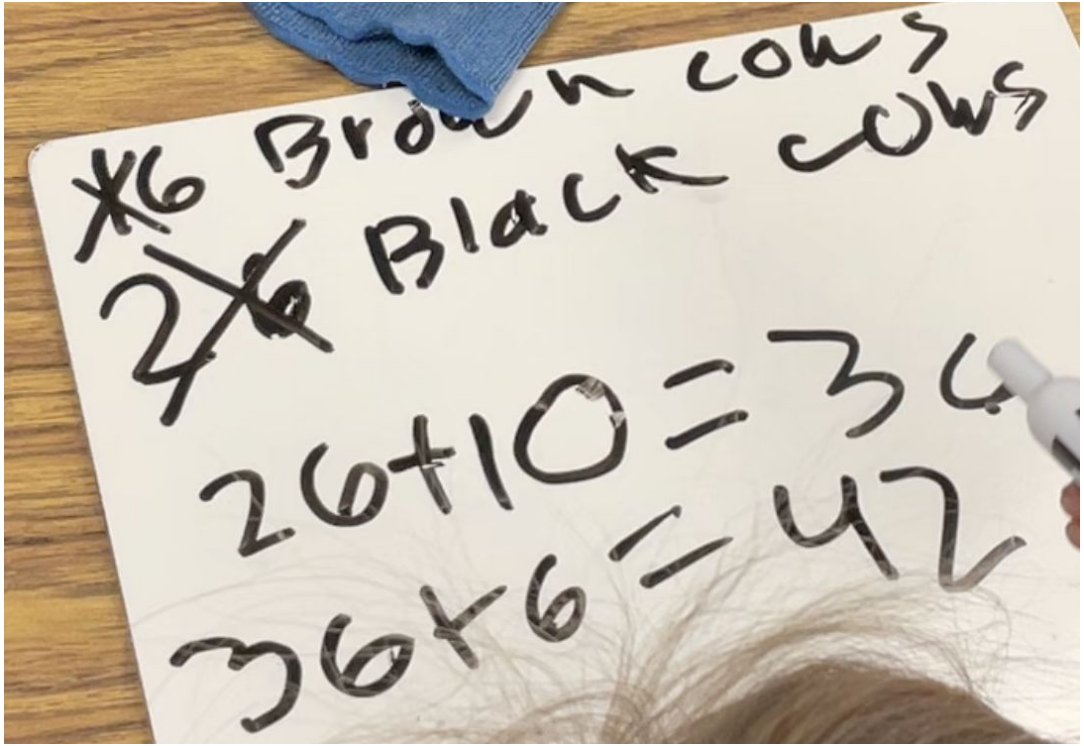
Hailey



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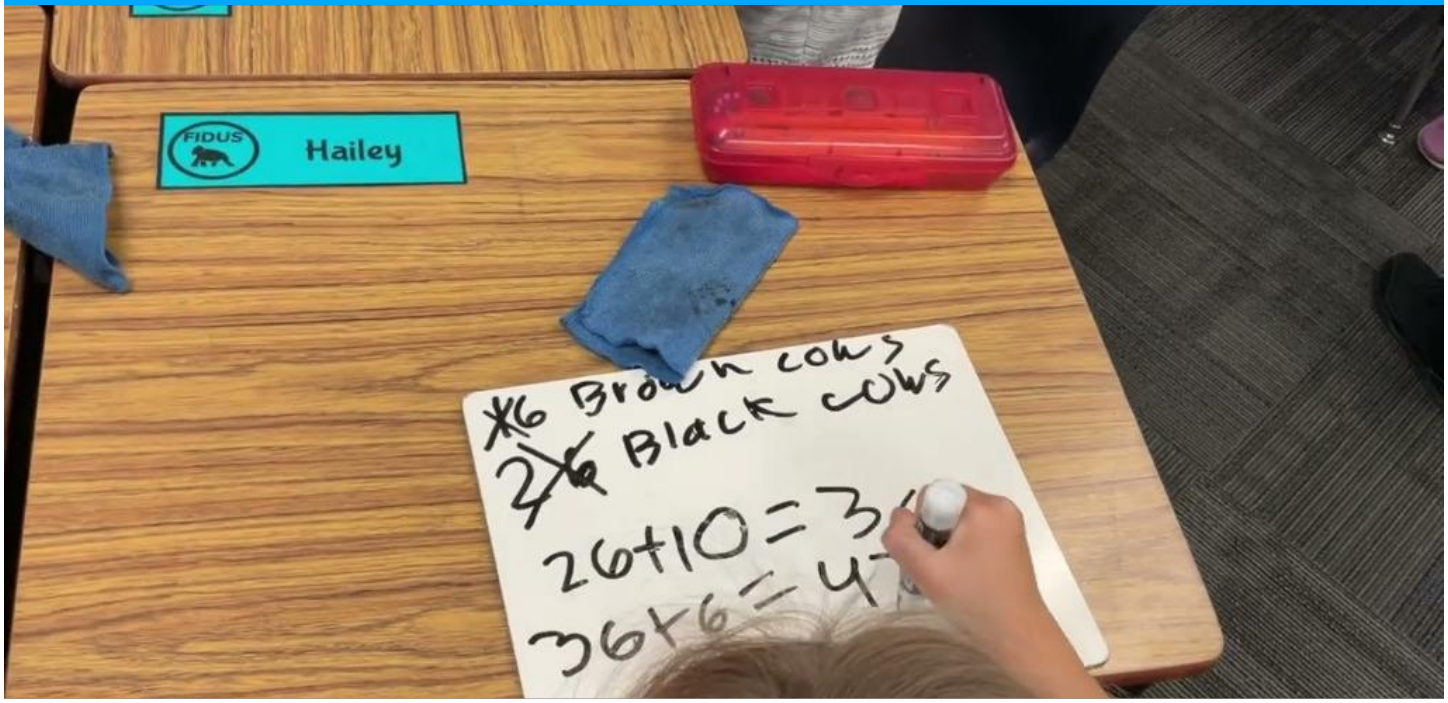

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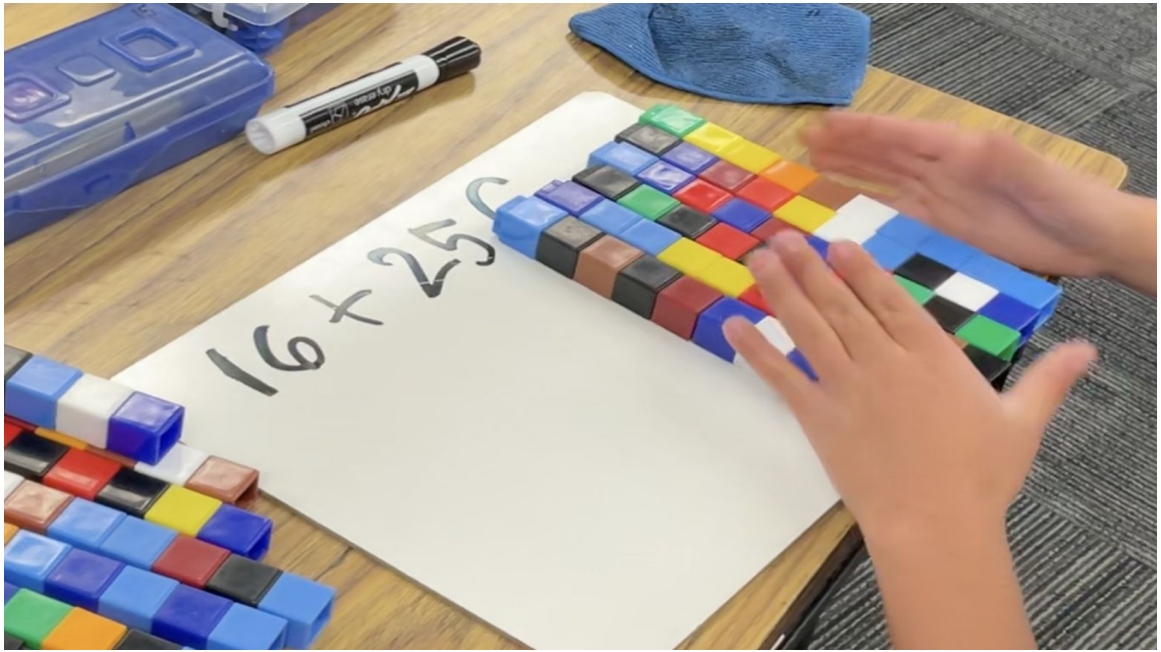

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Students Share Strategies in Whole Class Discussion

Evidence of Student Thinking

Ashley	
Evidence of student thinking:	What they understand: What I wonder about their understanding:

Knox	
Evidence of student thinking:	What they understand: What I wonder about their understanding:

Johnny	
Evidence of student thinking:	What they understand: What I wonder about their understanding:

Evidence of student thinking:

What they understand:

What I wonder about their understanding:

Learning Progressions & Learning Trajectories

Learning progressions and learning trajectories inform the pathways in which students learn content.

Sometimes the terms are used interchangeably, but depending on the source and context there are a few differences.

What Are Learning Progressions and Learning Trajectories?

"Learning progressions, progress maps, developmental continuums, and learning trajectories are all terms that have been used in the literature over the past decade. While many variations on the definition exist, the concept generally refers to research-based, descriptive continuums of how students develop and demonstrate deeper, broader, and more sophisticated understanding over time" (Hess, 2014).

Learning Progressions

Learning Progressions

"Learning progressions describe typical sequences of learning in specific areas or disciplines. ... while learning standards describe what a student should have learned by a specific stage in their education, learning progressions focus on the building blocks that contribute to mastering a particular skill. This developmental approach maps the progress of a student through stages of increasing knowledge, skills, and understanding"

(Kim & Scoular, 2017).

Learning Progressions and Standards

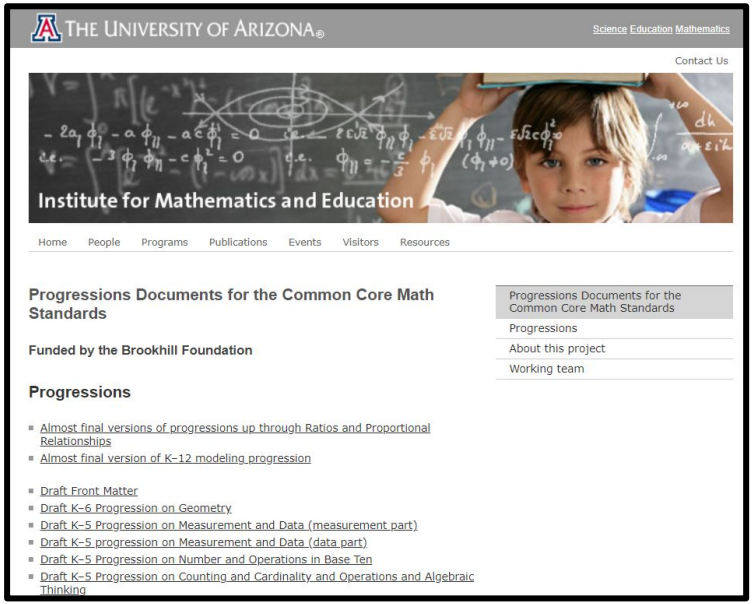
Learning progressions help us know the pathways students may take to learn concepts both within and across grade levels.

"The Standards in mathematics were built on progressions of topics across a number of grade levels, informed both by research on children's cognitive development and by the logical structure of mathematics"

(The University of Arizona, 2013).

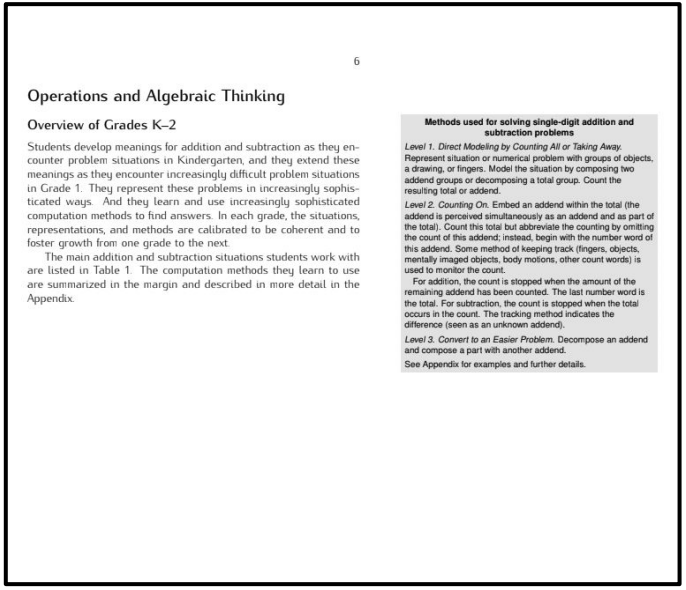
Progressions Documents

Narrative documents describing the progression of a topic across a number of grade levels



http://ime.math.arizona.edu/progressions/

Operations and Algebraic Thinking & Numbers and Operations in Base Ten



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Operations and Algebraic Thinking

Overview of Grades K–2

Students develop meanings for addition and subtraction as they encounter problem situations in Kindergarten, and they extend these meanings as they encounter increasingly difficult problem situations in Grade 1. They represent these problems in increasingly sophisticated ways. And they learn and use increasingly sophisticated computation methods to find answers. In each grade, the situations, representations, and methods are calibrated to be coherent and to foster growth from one grade to the next.

The main addition and subtraction situations students work with are listed in Table 1. The computation methods they learn to use are summarized in the margin and described in more detail in the Appendix.

Methods used for solving single-digit addition and subtraction problems

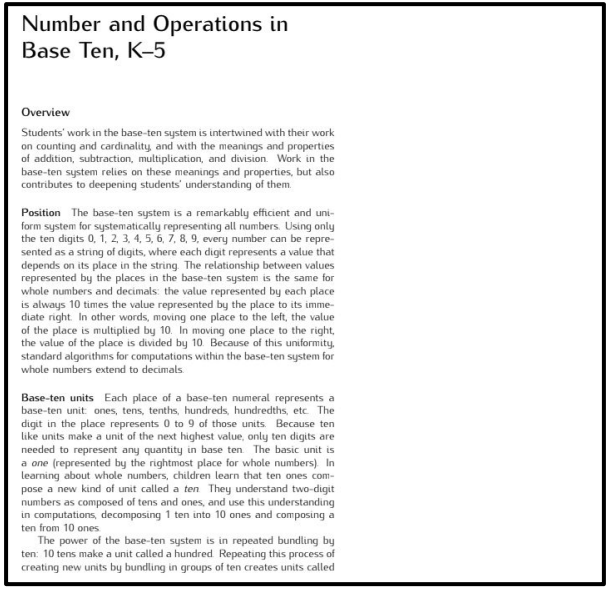
Level 1. Direct Modeling by Counting All or Taking Away. Represent situation or numerical problem with groups of objects, a drawing, or fingers. Model the situation by composing two addend groups or decomposing a total group. Count the resulting total or addend.

Level 2. Counting On. Embed an addend within the total (the addend is perceived simultaneously as an addend and as part of the total). Count this total but abbreviate the counting by omitting the count of this addend; instead, begin with the number word of this addend. Some method of keeping track (fingers, objects, mentally imaged objects, body motions, other count words) is used to monitor the count.

For addition, the count is stopped when the amount of the remaining addend has been counted. The last number word is the total. For subtraction, the count is stopped when the total occurs in the count. The tracking method indicates the difference (seen as an unknown addend).

Level 3. Convert to an Easier Problem. Decompose an addend and compose a part with another addend.

See Appendix for examples and further details.



Number and Operations in Base Ten, K–5

Overview

Students' work in the base-ten system is intertwined with their work on counting and cardinality, and with the meanings and properties of addition, subtraction, multiplication, and division. Work in the base-ten system relies on these meanings and properties, but also contributes to deepening students' understanding of them.

Position The base-ten system is a remarkably efficient and uniform system for systematically representing all numbers. Using only the ten digits 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, every number can be represented as a string of digits, where each digit represents a value that depends on its place in the string. The relationship between values represented by the places in the base-ten system is the same for whole numbers and decimals: the value represented by each place is always 10 times the value represented by the place to its immediate right. In other words, moving one place to the left, the value of the place is multiplied by 10. In moving one place to the right, the value of the place is divided by 10. Because of this uniformity, standard algorithms for computations within the base-ten system for whole numbers extend to decimals.

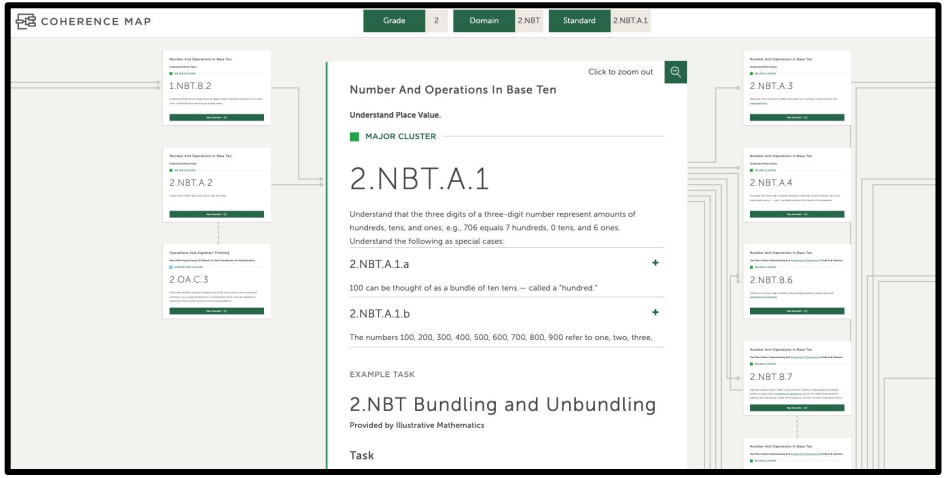
Base-ten units Each place of a base-ten numeral represents a base-ten unit: ones, tens, tenths, hundreds, hundredths, etc. The digit in the place represents 0 to 9 of those units. Because ten like units make a unit of the next highest value, only ten digits are needed to represent any quantity in base ten. The basic unit is a *one* (represented by the rightmost place for whole numbers). In learning about whole numbers, children learn that ten ones compose a new kind of unit called a *ten*. They understand two-digit numbers as composed of tens and ones, and use this understanding in computations, decomposing 1 ten into 10 ones and composing a ten from 10 ones.

The power of the base-ten system is in repeated bundling by ten: 10 tens make a unit called a hundred. Repeating this process of creating new units by bundling in groups of ten creates units called

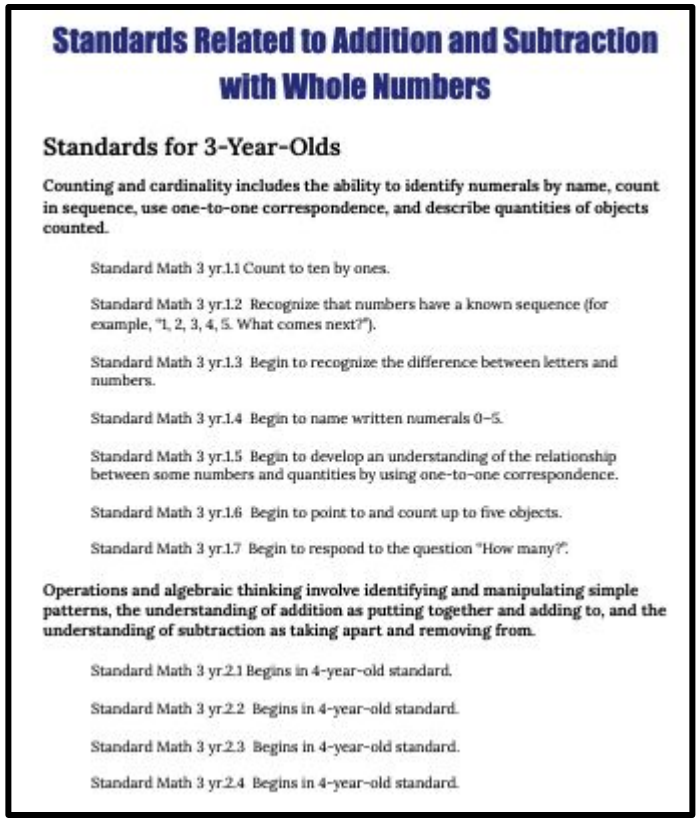
Coherence Map

Mathematics standards are not isolated concepts.

Standards relate to one another, both within and across grades. The Coherence Map illustrates the coherent structure that is fundamental to college- and career-ready standards.



https://achievethecore.org/coherence-map/



Standards Related to Addition and Subtraction with Whole Numbers

Standards for 3-Year-Olds

Counting and cardinality includes the ability to identify numerals by name, count in sequence, use one-to-one correspondence, and describe quantities of objects counted.

Standard Math 3 yr.1.1 Count to ten by ones.

Standard Math 3 yr.1.2 Recognize that numbers have a known sequence (for example, "1, 2, 3, 4, 5. What comes next?").

Standard Math 3 yr.1.3 Begin to recognize the difference between letters and numbers.

Standard Math 3 yr.1.4 Begin to name written numerals 0–5.

Standard Math 3 yr.1.5 Begin to develop an understanding of the relationship between some numbers and quantities by using one-to-one correspondence.

Standard Math 3 yr.1.6 Begin to point to and count up to five objects.

Standard Math 3 yr.1.7 Begin to respond to the question "How many?".

Operations and algebraic thinking involve identifying and manipulating simple patterns, the understanding of addition as putting together and adding to, and the understanding of subtraction as taking apart and removing from.

Standard Math 3 yr.2.1 Begins in 4-year-old standard.

Standard Math 3 yr.2.2 Begins in 4-year-old standard.

Standard Math 3 yr.2.3 Begins in 4-year-old standard.

Standard Math 3 yr.2.4 Begins in 4-year-old standard.

Number of Standards Per Grade Level

Age 3	7 (CC)
Age 4	7 (CC), 4 (OA)
K	5 (OA), 1 (NBT)
1	8 (OA), 3 (NBT)
2	2 (OA), 5 (NBT), 2 (MD)
3	1 (NBT)
4	1 (NBT)

*Standards are in the OA and NBT domains, as well as CC in PK & MD in Grade 2.

PK-2 Standards Progression in Addition and Subtraction

	Preschool	Kindergarten	First Grade	Second Grade
Word Problems	Begins in 4-year-old standard With prompting and support, solve addition and subtraction word problems created by the teacher using up to five concrete objects to represent the problem	Add/subtract within 10 Add to/Take from with result unknown Put together/Take apart with total unknown or both addends unknown	Add/subtract within 20 Add to/Take from with result unknown, change unknown, and start unknown Put together/Take apart with total unknown, both addends unknown, or one addend unknown Compare with difference unknown, bigger unknown, or smaller unknown	Add/subtract within 100 Add to/Take from all problem types; master start unknown Put together/Take apart all problem types Compare with all problem types; master bigger unknown and smaller unknown
Place Value	Begins in kindergarten standards	Compose and decompose numbers from 11 to 19 into ten ones and some further ones	Add within 100: - add a two-digit number and a one-digit number - add a two-digit number and a multiple of 10 - add tens and tens, ones and ones; - sometimes it is necessary to compose a ten Subtract multiples of 10 from multiples of 10 in the range 10-90 Mentally add and subtract 10	Fluently add and subtract within 100 using strategies Add and subtract within 1000: - add or subtract hundreds and hundreds, tens and tens, ones and ones; - sometimes it is necessary to compose or decompose tens or hundreds Mentally add and subtract 10 and 100

PK-2 Standards Progression in Addition and Subtraction

	Preschool	Kindergarten	First Grade	Second Grade
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Place Value	Begins in kindergarten standards	Compose and decompose numbers from 11 to 19 into ten ones and some further ones	Add within 100: - add a two-digit number and a one-digit number - add a two-digit number and a multiple of 10 - add tens and tens, ones and ones; - sometimes it is necessary to compose a ten Subtract multiples of 10 from multiples of 10 in the range 10-90 Mentally add and subtract 10	Fluently add and subtract within 100 using strategies Add and subtract within 1000: - add or subtract hundreds and hundreds, tens and tens, ones and ones; - sometimes it is necessary to compose or decompose tens or hundreds Mentally add and subtract 10 and 100

	Preschool	Kindergarten	First Grade	Second Grade
Fluency	Begins in kindergarten standards	Fluently add and subtract within 5	Fluently add and subtract within 10	Fluently add and subtract within 20 using mental strategies Know from memory all sums of two one-digit numbers Fluently add and subtract within 100 using strategies
Additional Concepts & Skills	Age 3 & 4: Identify numerals by name, count in sequence, use one-to-one correspondence, and describe quantities of objects counted Age 4: Use concrete objects, fingers, movement, and simple drawings	Use objects, fingers, mental images, drawings, sounds (e.g., claps), acting out situations, verbal explanations, expressions, or equations	Solve word problems with three addends (sum less than or equal to 20) Apply commutative and associative properties Understand the meaning of the equal sign Determine unknowns in equations	Add up to four two-digit numbers Relate addition and subtraction to length

References:

The University of Arizona. (2023). *Progressions Documents for the Common Core Math Standards: Draft K–5 Progression on Counting and Cardinality and Operations and Algebraic Thinking*. http://ime.math.arizona.edu/progressions/.

Utah State Board of Education. (2016). Utah Core State Standards for Mathematics. https://www.schools.utah.gov/curr/mathematics

Utah State Board of Education. (2020). Utah Core Standards Early Learning (Ages 3-5). https://www.schools.utah.gov/curr/preschool

Learning Trajectories

Learning Trajectories

"Although standards are essential in identifying what topics to teach and when to teach them, they offer little insight into how to teach those topics. Learning trajectories (LTs) are empirically grounded descriptions of how students' reasoning evolves from less to more sophisticated. They can provide deep insight into how to teach topics during a single grade as well as how topics develop and evolve across the grades."

(Confrey, Shah, & Maloney, 2022).

Learning Trajectories

“We liken LTs to a climbing wall rather than a ladder. … A ladder implies that students proceed uniformly through strictly prerequisite levels. A climbing wall assumes climbers move upward from a variety of starting points through multiple paths. LT levels, when envisioned as handholds, footholds, and obstacles, make student thinking visible and predictable, though probabilistic" (Confrey, Shah, & Maloney, 2022).

Learning Trajectories

– – –

"Children follow natural developmental progressions in learning. Curriculum research has revealed sequences of activities that are effective in guiding children through these levels of thinking. These developmental paths are the basis for the learning trajectories. ...

Learning trajectories allow teachers to build the mathematics of children – the thinking of children as it develops naturally."

(Clements & Sarama, 2017/2019).

Learning and Teaching with Learning Trajectories

Differentiated instruction for early mathematics instruction is supported through understanding and use of Learning Trajectories - a goal, a developmental progression, and learning opportunities.

Directions:

- Go to learningtrajectories.org to access Learning & Teaching with Learning Trajectories
- Sign up using your email address
- Select “EXPLORE LTS”
- Explore the resources for the following areas:
  - Adding/Subtracting

Learning Trajectories in Addition and Subtraction

Learning Trajectories	Numerical Parameters	Strategies and Representations
Arithmetic Senser: Foundations Preverbal +/- Small Number +/- Find Result +/- Make It N Find Change +/- Counting Strategies +/- Part-Whole +/- Numbers-in-Numbers +/- Deriver +/- (Adding/Subtracting) Problem Solver +/- (Adding/Subtracting) Multidigit +/- (Adding/Subtracting) See details at: https://www.learningtrajectories.org/math/learning-trajectories/adding-subtracting	-Add/subtract within 5 -Add/subtract within 10 -Add/subtract within 20 -Add a two-digit number and a one-digit number -Add a two-digit number and a multiple of 10 -Add two-digit numbers without composing a 10 -Add two-digit numbers with composing a 10 -Subtract multiples of 10 from multiples of 10 in the range 10-90 -Subtract a two-digit number and a one-digit number -Subtract a two-digit number and a multiple of 10 -Subtract two-digit numbers without decomposing a 10 -Subtract two-digit numbers with decomposing a 10 -Add three digit numbers without composing tens or hundreds -Subtract three digit numbers without decomposing tens or hundreds -Add three digit numbers with composing tens or hundreds -Subtract three digit numbers with decomposing tens or hundreds Add/subtract multi-digit numbers	Direct Modeling: Counting by ones Direct Modeling: Using base-ten models Invented Strategies: Supported by written recordings Invented Strategies: Using mental methods Representations: -Open number lines -Bar diagrams -Expanded form Addition Strategies: -Add tens, add ones, then combine -Add on tens, then add of ones -Move some to make a ten -Use friendly numbers to compensate Subtraction strategies: -Take tens from the tens, then subtract ones -Take away tens, then ones Standard Algorithms: Prove that it produces a correct answer.

Learning Progressions & Learning Trajectories

Learning Progressions and Learning Trajectories

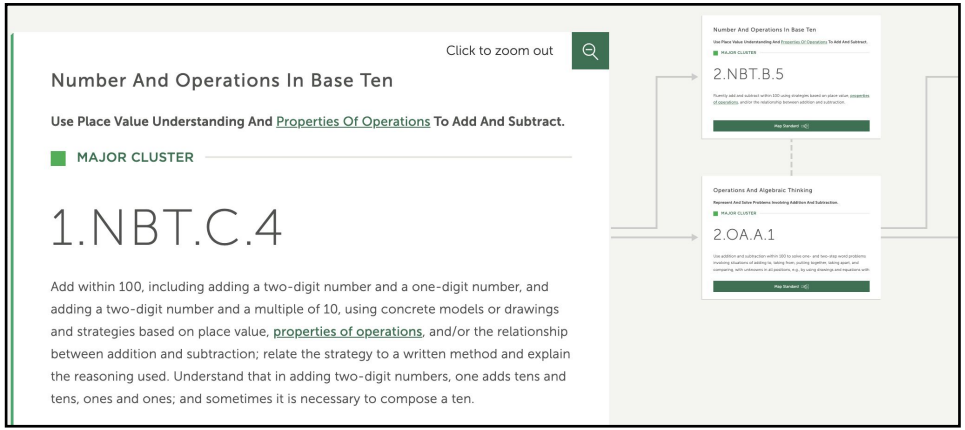
Definitions:

Learning progressions describe how big ideas advance across grade levels and the way standards connect over time. They tell us *when* students learn concepts.

Learning trajectories refer to the pathways children naturally take to develop mathematical understanding. They tell us *how* students learn concepts.

Learning Progressions and Learning Trajectories

Examples:



https://achievethecore.org/coherence-map

https://www.learningtrajectories.org

Learning Progressions and Learning Trajectories

Applications:

- Planning for whole class instruction
- Planning for grade level instruction
- Knowing the flow of big ideas across time

- Planning for differentiated instruction
- Planning for skill-based instruction
- Knowing the flow of developing individual skills

Intervention Strategies

Intervention Strategies

What strategies have you tried?

What are some things that work for intervention?

Intervention Strategies

- Learning Progressions
- Learning Trajectories
- Alignment of Intervention to Core Instruction
- Context
- Mathematical Language
- Representations

Reference: Institute of Education Sciences: What Works Clearinghouse. (2021, March). WWC: *Assisting students struggling with mathematics: Intervention in the elementary grades*. https://ies.ed.gov/ncee/wwc/PracticeGuide/26.

Learning Progressions:

Use learning progressions to know when to teach concepts in relation to other concepts and grade level expectations.

Learning Trajectories:

Use learning trajectories to know how children develop understanding of concepts.

Alignment of Intervention to Core Instruction:

Systematically ensure that learning in intervention directly supports learning in Tier 1 instruction.

Context:

– – –

Present mathematical situations using word problems students can understand to deepen students' mathematical understanding and support their capacity to apply mathematical ideas.

Mathematical Language:

– – –

Allow students to naturally use informal language as they develop understanding of concepts. Once concepts are surfaced, teach clear and concise academic language.

Representations:

– – –

Provide students with physical tools to solve mathematical problems and connect those physical models to pictorial and other representations. Regularly include number lines as representations.

Strategies

Learning Progressions:
Use learning progressions to know when to teach concepts in relation to other concepts and grade level expectations.

Learning Trajectories:
Use learning trajectories to know how children develop understanding of concepts.

Alignment of Intervention to Core Instruction:
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Representations:
Provide students with physical tools to solve mathematical problems and connect those physical models to pictorial and other

Intervention in Action

Intervention in Action

– – –

Record the following:

- Evidence of Student Thinking
- Evidence of Use of Learning Progressions/Trajectories
- Evidence of Use of Intervention Strategies

Adding Multiples of 10

Video

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Finn

Video

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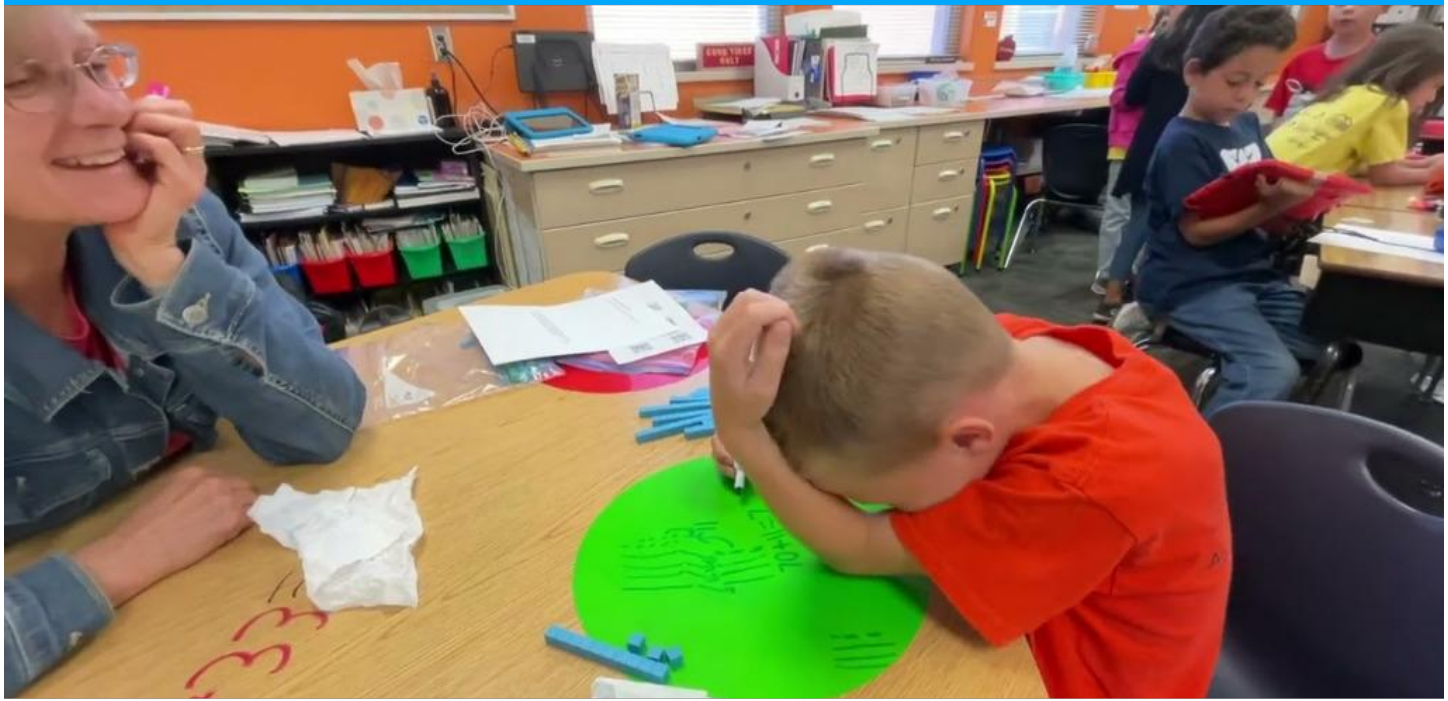
Parker

Video



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Intervention in Action

– – –

What evidence did you see?

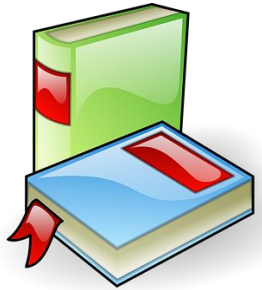
- Evidence of Student Thinking
- Evidence of Use of Learning Progressions/Trajectories
- Evidence of Use of Intervention Strategies

Closing and Reflection

Thank you to Mrs. Struthers and her class!

Second Grade
Meadow Elementary
Lehi, UT

Resources and References



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Closing and Reflection

What do you want to remember from the session?

Questions and feedback:

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